

To teach mathematics is to guide a discovery. Learning mathematics is essentially discovering mathematical *ideas*, such as geometric intuition, simplification and use of symmetry. Mathematical ideas take place in a mathematician's mind, and thus nobody can perfectly instill these on the mind of another. Nonetheless, an instructor can facilitate the learning process, helping a student *recreate* in his or her mind the beautiful idea presented in the class. In time, I see that my students come to express these ideas themselves in proper forms of logical statements, such as axioms, theorems, proofs, and computations.

A crucial role of an instructor is to cultivate an atmosphere in the class which allows this collective process of discovery. After making administrative announcements, I begin each class by describing the day's "Learning Objectives". For example, if a class is on normal distribution in probability, I clearly write the topics such as definition, expectation, standard deviation and approximation of binomial distribution. Then I briefly recall definitions and theorems under the heading "Recall". Usually, this includes the material from the beginning of the chapter. Students are often surprised to learn how concise the material may be summarized in such a systematic way. The next part of the class is "Motivation". In this part, I present a simple but intellectually stimulating question, which is easy to understand but not so straightforward to answer. In the case of normal distribution, I would give a two-part question: the first part is to compute the probability that at most 3 heads show up when 4 coins are tossed; the second part is to compare it with the probability of having at most 300 heads when 400 coins are tossed. The first part is answered using the techniques introduced in previous classes. The content of the lecture then answers the second part of the question, as well as generalizing it to other types of events. This way, a new idea becomes a lot more intuitive and straightforward generalization of previous material.

For the main part of a lecture, I state definitions and theorems in logically coherent and concise sentences, which at first may be difficult for the students to fully understand. Then I spend a considerable amount of time to help students to discover the ideas behind those sentences. I always start from the very basic examples and build their understanding to the level of the most general cases. Finally, I return to the original definitions and theorems. The great difference in such an approach is that not only are students capable of solving problems algorithmically and systematically, they now also understand what it is that they are computing.

Class interaction is a key indicator measuring the success of each class. I maintain eye-contact carefully, ask soft, qualitative questions that can be answered by most students, and pause for any feedback. Whenever I get asked a question, I try to commend the student for any original idea that the question implies, regardless of whether the question makes mathematical sense or not. If the question is about the topic that was just discussed, I rephrase the explanation with different sentences or examples. I sometimes quote results from the previous lectures, but I never blindly assume that the audience remembers them. Other than regular office hours, I contact students who have a hard time with the course material or exams. I also highly value student-student interaction, inside and outside of class. I strongly encourage students to group together for homework and exam preparation. Critical thinking and true discovery can be fully developed through discussion of the material that they are learning.

I have taught 15 courses over the past eight years at Yale University and at the University of Texas at Austin. These include summer courses, multi-variable calculus for economics graduate students, number theory, probability and discrete mathematics. Calculus classes varied from small (less than ten students) to large (more than a hundred). For all these courses, my philosophy of teaching and the structure of a lecture have always been consistent. To aid motivation and discovery of students, I find that visualization is a very powerful tool. This applies to almost all parts of mathematics, such as geometry, calculus and even number theory, not least because I am a geometric topologist. In addition to color chalks and markers, I make great use of computer-aided graphics softwares such as Mathematica[®] both in class and in homework assignment.

In the Spring semester 2010, I will teach a graduate course, *Topics in Geometric Group Theory*. Aimed at beginning graduate students, this course will discuss basic notions of geometric group theory and eventually lead to the study of cube complexes. In mathematics, even very familiar examples can be viewed with different perspectives to effectively introduce new insights. For example, I plan to examine the infinite cyclic group extensively in order to embark on discussion of Cayley graphs and quasi-isometries. Advising and interacting with students in a Ph.D. or master's program have been greatly inspiring to me and often revealed leads to my research, not to mention theirs.

A good guide does not lift and carry the traveler. A good guide walks one foot ahead, encouraging the traveler onwards and clearly pointing to the destination. Teaching mathematics is not conveying an algorithm of solving problems. Rather, teaching mathematics is about leading the students to discover marvelous ideas that the logical statements of mathematics imply. To do that, the teacher must set specific goals for each class, and help the students achieve attainable goals through well-organized lectures. Learning a mathematical topic is complete only after the students recreates the ideas in the own mind and appreciates the meaningfulness of the topic.